AN OBSERVATIONAL TEST OF THEORIES OF NEUTRON STAR COOLING[*]

The purpose of this note is to point out that theories describing the cooling of neutron stars are subject to an observational test should a neutron star ever be detected. The existence of such a test is particularly important because theories of neutron star cooling by neutrino emission necessarily involve the use of at least a modicum of strong interaction theory in a density region $(\rho > \rho_{nuclear})$ which cannot be studied in the laboratory. In order to prevent the investigation of neutron stars from degenerating into a philosophical discussion, it is necessary to concentrate on those aspects of the theory which are at least in principle connected with observation.

The basic idea of the test we propose is that the photon luminosity of a neutron star should decrease more rapidly if neutrino energy losses are present than if the only cooling mechanism were photon emission. In particular, a measurement of $\varphi^{-1} d\varphi/dt$, the time derivative of the logarithm of the photon flux, should be sufficient to determine whether or not the rate of cooling by neutrino emission is, as predicted (Bahcall and Wolf 1965a) for most neutron star temperatures that are observationally accessible, large compared to the rate of photon emission. Fortunately, the theoretical expression for $\varphi^{-1} d\varphi/dt$ does not depend critically on the detailed properties of the neutron star model assumed.

In order to determine the order of magnitude of the effects due to neutrino emission, we have computed the cooling times of a typical neutron star (mass $\sim 1\ M_{\odot}$, density $\sim \rho_{nuclear}$) assuming the existence of two different mechanisms for neutrino emission and a degenerate Fermi-Dirac spectrum of

---

energies for each of the relevant kinds of particles present in a neutron
star. The equations used in these calculations are:

$$\frac{dU}{dt} = (L_\nu + L_\gamma)$$ (1)

and

$$T \approx \alpha \times 10^{+2} T_e .$$ (2)

Here U is the total thermal energy of the degenerate core, L_ν and L_γ are,
respectively, the energy emitted per unit of time in the form of neutrinos
and photons, and T, T_e are, respectively, the central and effective (i.e.,
surface) temperatures. If one assumes an independent particle model, one can
easily obtain the following approximate expression for U:

$$U \approx 5 \times 10^{+47} T_9^{+2} (\rho/\rho_{nucl})^{-2/3} (M/M_\odot) \text{ erg,}$$ (3)

where T_9 is the core temperature in units of 10^{+9} °K. The appropriate ex-
pressions for L_ν have been given previously (Bahcall and Wolf 1965a). We
assume that the photon luminosity is that of a blackbody (Morton 1964a,b;
Chiu and Salpeter 1964) and use equation (2) to relate the computed interior
cooling rates to the surface properties of the star. The coefficient, α, in
equation (2) is about equal to two (see the second column of Table 1); it has
been chosen so that equation (2) approximately reproduces the results of de-
tailed model calculations by Cameron and Tsuruta (Tsuruta 1964, Cameron and
Tsuruta 1965) for temperatures in the observationally accessible region
($T_e \approx 10^{+6}$ to 10^{+7} °K, $\lambda_{max} \approx 1$ to 30 A°) and for a density equal to nuclear
density. We believe that there are no reliable models for densities much
greater than nuclear densities.

If the dominant energy loss occurs by the generalized neutrino URCA
process (Chiu and Salpeter 1964; Finzi 1965; Bahcall and Wolf 1965a) then one
can show with equations (1), (3) and our previously published cooling rates

that:

$$\Delta t \ (\text{URCA}) \approx 5 \times 10^{+1} \ T_9(f)^{-6} \left(1 - [T_9(f)/T_9(i)]^{+6}\right) (\rho/\rho_{\text{nucl}})^{+1/3} \ \text{yrs}, \quad (4)$$

where Δt is the time required for a neutron star to cool from an initial central temperature, $T(i)$, to a final central temperature, $T(f)$. Similarly, if the dominant energy loss occurs via pion beta decay, then:

$$\Delta t \ (\text{PIONS}) \approx 2.5 \times 10^{-6} \ T_9(f)^{-4} \left(1 - [T_9(f)/T_9(i)]^{+4}\right) (\rho/\rho_{\text{nucl}})^{+4/3} \ \text{yrs}. \quad (5)$$

If the dominant energy loss occurs by photon emission, then

$$\Delta t \ (\text{Photons}) = 2\alpha^2 \times 10^{+3} \ T_e'(f)^{-2} \left(1 - \left[\frac{T_e(i)}{T_e(f)}\right]^{+2}\right) \ \text{yrs}, \quad (6)$$

where T_e' is expressed in units of $10^{+7} \ ^\circ K$. In deriving equation (6), we have made use of the fact that the quantity α is approximately constant for small temperature (or time) intervals.

In Table 1, we compare neutrino and photon cooling times for a neutron star with mass equal to one solar mass and radius equal to 10 Km. Note that the URCA process is much faster than photon cooling for $T_e \gtrsim 4 \times 10^6$ ($\lambda_{\text{max}} \lesssim 12 \ A^\circ$). Pionic neutrino cooling is dominant, if present at all, for any $T_e \gtrsim 10^3 \ ^\circ K$. The cooling times given in column 5 of Table 1 are so short (seconds to days) that one could not hope to observe a neutron star by x-ray emission if the pionic cooling mode is indeed present. Thus the possibility of making macroscopic astronomical observations on neutron stars depends critically on microscopic properties of strongly interacting particles, properties that are not well understood at present.

We have investigated a number of other possible neutrino cooling processes (Bahcall and Wolf 1965b) and, so far, the URCA and pionic processes are the fastest ones that we have found. If other even faster cooling processes are

discovered, then cooling times for these processes can also be computed in the manner indicated above. The predicted effects would then be even larger than the already striking differences, illustrated in Table 1, between photon and neutrino cooling.

The observational test we propose is to measure the photon flux in two or more wavelength regions (thus determining the effective temperature) at two or more different times. One can then form the logarithmic derivative of the total photon flux, φ, with respect to time; the predicted value of this quantity is given below:

$$\frac{1}{\varphi}\frac{\Delta\varphi}{\Delta t}_{\text{years}} = \alpha^{-2} \times 10^{-3} \left(\frac{M}{M_\odot}\right)^{1/3} \times$$

$$T_e'^2 \left[1 + 10\alpha^8 \left(\frac{M}{M_\odot}\frac{\rho_{\text{nucl}}}{\rho}\right)^{1/3} T_e'^4 + 10^{+9}\left(\frac{n_\pi}{n_n}\right)\alpha^6 T_e'^2 \left(\frac{M}{M_\odot}\right)^{1/3}\left(\frac{\rho}{\rho_{\text{nucl}}}\right)^{-4/3} \right], \quad (7)$$

where (n_π/n_n) is the ratio of the pion number density to the nucleon number density. (We assumed $n_\pi/n_n \approx 1/3$ in computing Table 1.) The three terms in brackets in equation (7) are, respectively, the contributions of photons, URCA neutrinos, and pionic neutrinos.

The advantage in using relation (7) to test theories of neutron star cooling is that this predicted relation does not depend on the distance to the source and only depends rather weakly on the properties (such as mass and density) of the neutron star model. It would be useful to compute $\varphi^{-1}(\Delta\varphi/\Delta t)$ for other suggested models of the observed x-ray sources.

April 8, 1965

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA

JOHN N. BAHCALL

RICHARD A. WOLF

TABLE 1

COMPARISON OF PHOTON AND NEUTRINO COOLING TIMES

The values of $\alpha = 10^{-2}\, T/T_e$ were taken from the atmospheric calculations of Tsuruta (1964).

T_e ($^{\circ}$K)	α	Δt (Photons) (yrs)	Δt (URCA) (yrs)	Δt (PIONS) (yrs)
$1.6 \times 10^{+7}$	2.3			
		$3 \times 10^{+3}$	10^{-1}	5×10^{-8}
$1.2 \times 10^{+7}$	2.0			
		$3 \times 10^{+3}$	1	10^{-7}
$9.4 \times 10^{+6}$	1.85			
		$7 \times 10^{+3}$	10^{+1}	10^{-6}
$6.7 \times 10^{+6}$	1.7			
		$9 \times 10^{+3}$	10^{+2}	5×10^{-6}
$5.1 \times 10^{+6}$	1.6			
		$9 \times 10^{+3}$	$5 \times 10^{+2}$	5×10^{-6}
$4.3 \times 10^{+6}$	1.6			
		$8 \times 10^{+4}$	10^{+5}	5×10^{-4}
$2.0 \times 10^{+6}$	1.3			
		$4 \times 10^{+5}$	10^{+8}	5×10^{-2}
$7.4 \times 10^{+5}$	1.0			

REFERENCES

Bahcall, J. N. and Wolf, R. A. 1965a, _Phys. Rev. Letters,_ 14, 343.

_____ 1965b, _Phys. Rev.,_ to be published. (It is

shown in this paper that the reaction $x^- + n \to e^- + \bar{v}_e + n'$ occurs with

essentially the same rate as that given for $x^- + n \to \mu^- + \bar{v}_\mu + n'$, i.e.,

no factor of $(m_e/m_\mu)^{+2}$ occurs.)

Chiu, H-Y. and Salpeter, E. E. 1964, _Phys. Rev. Letters,_ 12, 413.

Cameron, A. G. W. and Tsuruta, S. 1965, to be published.

Finzi, A. 1965, _Phys. Rev.,_ 137, B472.

Morton, D. C. 1964a, _Nature,_ 201, 1308.

_____ 1964b, _Ap. J.,_ 140, 460. (This article, which discusses photon

cooling, contains the valuable remark that the small change in photon

luminosity due to blackbody cooling should be measurable in the near

future with well-calibrated detectors.)

Tsuruta, A. 1964, Thesis, Columbia University (unpublished). Equation (2) of

our note is based on Table 26 of this thesis.